

Scale invariant gravity and the quasi-static universe

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Abstract. We highlight the fact that the lack of scale invariance in the gravitational field equations of General Relativity results from the underlying assumption that the appropriate scale for the gravitational force should be linked to the atomic scale. We show that many of the problems associated with cosmology and quantum gravity follow directly from this assumption. An alternative scale invariant paradigm is proposed, in which the appropriate scale for General Relativity takes the Universe as its baseline, and the gravitational force does not have any fixed relationship to forces that apply on the atomic scale. It is shown that this gives rise to a quasi-static universe, and that the predicted behaviour of this model can resolve most of the problems associated with the standard Big Bang model. The replacement of Newton's gravitational constant in the quasi-static model by a scale-dependent renormalisation factor is also able to account for a number of astronomical observations that would otherwise require ad-hoc explanations. Some of the implications of scale invariant gravity for Planck scale physics, quantum cosmology, and the nature of time are discussed.

1. Introduction

Einstein's General Theory of Relativity has proved to be one of the most successful and enduring theories in physics, and its predictions have been verified in numerous experiments. However, it stands alone amongst field theories in that it is not scale invariant. For example, the differential form of Maxwell's equations, which elegantly describe the electromagnetic field, do not define any intrinsic scale. Conversely, Einstein's field equations, which describe the way that matter curves spacetime, are linked to an apparently arbitrary scale determined by the Newtonian gravitational constant, G .

This can be verified by inspection of the standard Einstein-Hilbert action for General Relativity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_M(\phi) \quad (1)$$

In the case of a universe without matter, only the curvature term in the action will be applicable. This can readily be made scale invariant by removing the $16\pi G$ factor using the transformation $g'_{\alpha\beta} \rightarrow \Omega^2 g_{\alpha\beta}$, $R' \rightarrow \Omega^{-2} R$, where Ω is a constant. Similarly, if the matter Lagrangian term is considered in isolation, it can be shown that any constituent fields that obey the Dirac equation, for example the electromagnetic field, will be also be invariant with respect to a conformal transformation. However, when the gravitational and matter terms are combined as in (1), the resulting action is no longer scale invariant.

The scale dependency inherent in General Relativity leads to an apparently fundamental scale, defined by the Planck units:

Planck Length

$$L_P \equiv \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35} m \quad (2)$$

Planck Time

$$T_P \equiv \frac{L_P}{c} \simeq 10^{-43} s \quad (3)$$

Planck Mass

$$M_P \equiv \sqrt{\frac{\hbar c}{G}} \simeq 10^{-8} kg \quad (4)$$

No theory of Quantum Gravity is yet able to explain the wide disparity between the Planck scale and the atomic scales that govern the everyday world we inhabit. Whilst the magnitude of the Planck length and time scales, at $\simeq 10^{-20}$ of the corresponding atomic length and time scales, are just about reconcilable with the concept of an evolving quantum Universe, it is difficult to account for the fact that the Planck mass is $\simeq 10^{20}$ times larger than the proton mass.

The remainder of this introductory section provides a brief historical overview of the development of alternative formulations of gravity. In Section 2 we review some of the main problems that exist in cosmology and quantum gravity, and show that these follow as a direct consequence of the lack of scale invariance in General Relativity. Section 3 introduces an alternative approach to scale-invariant gravity, based on the redefinition of the gravitational field equations using cosmological units. It is shown that the resulting dynamical equations describe a universe that can be either static or expanding, depending on the reference frame of the observer. In Section 4, some of the consequences of the scale invariant version of General Relativity described in Section 3 are examined. It is shown that the modified dynamical behaviour can satisfactorily account for the problems associated with Big Bang cosmology. Section 5 concludes by examining some of the implications of abandoning the Planck scale for the study of quantum gravity.

Numerous attempts have been made to develop a theory of gravitation that is scale invariant, and yet retains the key properties of General Relativity, such as the principle of general covariance. These can be classified according to their main approach to the problem:

- Adding a scalar field to the standard Einstein-Hilbert action
- Resorting to a higher order description of gravity in place of the Riemann tensor of General Relativity
- Modifying the behaviour of the fields that constitute the matter Lagrangian

There are also a number of hybrid theories that incorporate more than one of the above approaches.

Scalar-Tensor theories Probably the best known of this class of theories is the Brans-Dicke theory [1]. The motivation for the development of this variant of General Relativity was to create a theory of gravity that explicitly incorporated Mach's principle, and the Dirac Large Number Hypothesis (LNH) [28], whilst still retaining the essential

symmetric and divergence-free nature of the original Einstein field equations. Although the Brans-Dicke theory does not directly address the issue of scale invariance, it is of particular interest in the historical development of this subject as it derives from the underlying Mach's principle: that inertial mass results from the gravitational interaction between matter and all other matter in the universe, and that the gravitational constant is itself determined by the matter distribution in the Universe, so that $GM/Rc^2 \sim 1$. Accordingly, either G or M must vary over time as R increases. In the introduction to [1], the authors remark on the fact that the metric tensor is dependent on the choice of units used in any particular representation of the gravitational field equations. The units chosen in General Relativity are such that nucleons have physical properties that are independent of location (and time).

In the Brans-Dicke theory the gravitational constant in the Einstein field equation is replaced by a scalar field $\phi \sim 1/G$. In order to preserve the divergence-free nature of the LHS of the field equation, such that $T^{\mu\nu}_{;\mu} = 0$, it is necessary to introduce various derivatives of the scalar field, together with a coupling constant ω , to give the Brans-Dicke action

$$S = \int d^4x \sqrt{-g} R \left[\frac{\phi R}{16\pi} \mathcal{L}_m - \frac{\omega}{\phi} \nabla_\mu \phi \nabla^\mu \phi \right] \quad (5)$$

Varying (5) with respect to the metric gives the modified field equation for Brans-Dicke gravity

$$G_{\mu\nu} = \frac{8\pi}{\phi} T_{\mu\nu} - \frac{\omega^2}{\phi} (\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi) - \frac{1}{\phi} (\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \partial^\alpha \partial_\alpha \phi) \quad (6)$$

It is worth noting that the scalar field ϕ in the Brans-Dicke theory is analogous to the dilaton field ϕ that arises in formulations of gravity derived from string theory.

Other applications of the scalar-tensor approach include the Conformal General Relativity model proposed in [2], and the Non-Gravitating Vacuum Energy model (NGVE) [3]. The former incorporates a conformal invariant $\hat{g}_{\mu\nu} = W^2 g_{\mu\nu}$, where W is a dilaton scalar field described by the Penrose-Chernikov-Tagirov (PCT) action

$$I = - \int d^4x (-\hat{g})^{1/2} R(\hat{g})/6 \quad (7)$$

The NGVE model features a scalar density in addition to the standard Lagrangian of General Relativity.

Higher order gravity theories The standard theory of General Relativity is based on the second order Einstein-Hilbert action of (1), which is formed from the Riemann curvature tensor. An alternative form of gravity was proposed by Weyl in 1918, in which the Einstein-Hilbert action is replaced with a conformal invariant fourth order action

$$I_W = -\alpha \int d^4x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \quad (8)$$

where $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor and α is a purely dimensionless coefficient. This formulation was used as the basis for the Conformal Cosmology model developed by Mannheim [4, 5]. Conformal gravity possesses no intrinsic fundamental length scale. The gravitational equations must be fourth order, so standard the Einstein equation is replaced by

$$4\alpha W_{\mu\nu} = 4\alpha(W_{\mu\nu}^{(2)} - W_{\mu\nu}^{(1)}/3) = T_{\mu\nu} \quad (9)$$

where $W_{\mu\nu}$ is given by

$$\begin{aligned} W_{\mu\nu}^{(1)} &= 2g_{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta} - 2(R^\alpha{}_\alpha)_{;\mu;\nu} - 2R^\alpha{}_\alpha R_{\mu\nu} + g_{\mu\nu}(R^\alpha{}_\alpha)^2/2 \\ W_{\mu\nu}^{(2)} &= g_{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/2 + R_{\mu\nu}^{;\beta}{}_{;\beta} - R_{\mu}{}^\beta{}_{;\nu;\beta} - R_{\nu}{}^\beta{}_{;\mu;\beta} - 2R_{\mu\beta}R_{\nu}{}^\beta + g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} \end{aligned} \quad (10)$$

Modified matter Lagrangian theories The most notable theory of this type is the steady-state model of Hoyle and Narlikar [6]. Essentially, this involves particle masses increasing as the Universe expands, and the existence of a scalar field (the C-field) that is capable of continuously creating new particles. The creation of new matter exactly balances the decrease in matter density resulting from the expansion of the Universe in such a way that the energy density in the Universe remains constant, and the gravitational equations become scale invariant.

Hybrid theories An example of a theory that combines two of the above approaches is the Scale-Covariant Theory of Gravitation proposed by Canuto et al [7, 8]. This incorporates an additional tensor term, together with a time dependent scalar factor, in the gravitational action. The action is then fourth order, and has similar scale invariant properties to the Weyl action.

As a generalisation, it can be said that all three of these approaches have the ability to address specific problems associated with the lack of scale invariance in General Relativity, but only at the expense of abandoning the elegance of Einstein's original theory. Furthermore, none of these approaches is able to provide a comprehensive framework that resolves all the issues identified in the following section.

2. The Planck scale crisis

The objective of this section is to illustrate how the lack of scale invariance in the field equations of General Relativity leads ultimately to a range of cosmological problems, and a crisis at the Planck scale. The consequences of this scale invariance can be broadly broken down into four areas:

- Cosmological dynamics
- Singularities
- Energy conservation
- Time

2.1. Cosmic dynamics

The dynamical behaviour of the Universe is conventionally determined by applying the Einstein field equations to the Robinson-Walker metric in order to derive the Friedman equations. Although this treatment is quite standard, and described in numerous references, e.g. [9], the main steps are shown here in order to highlight the point at which the assumption is made that results in the Planck scale crisis.

2.1.1. The Friedman equations Starting with the Einstein equation, derived from applying the variation principle to the Einstein-Hilbert action of (1)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (12)$$

and the Robinson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (13)$$

we can derive the first two components of the Einstein tensor

$$G_{00} = \frac{3\dot{a}^2}{a^2 c^2} + \frac{3k}{a^2} \quad (14)$$

$$G_{11} = - \frac{k + 2a\ddot{a}/c^2 + \dot{a}^2/c^2}{1 - kr^2} \quad (15)$$

The equations of motion are derived by combining (14,15) with the corresponding 00 and 11 components of the stress-energy tensor to give

$$\frac{3\dot{a}^2}{a^2} + \frac{3k}{a^2} = 8\pi G\rho + c^2 \Lambda \quad (16)$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi Gp - c^2 \Lambda \quad (17)$$

It is at this point that the assumption inherent in equation (12) manifests itself. Implicitly, in applying the Einstein equation, it is assumed that the energy density ρ is measured in conventional units that are defined in terms of the atomic properties of matter, e.g. MKS units. Similarly, it is assumed that the scale of the spacetime curvature induced by this stress-energy is determined by the Newtonian gravitational constant G , expressed in the same units. By expressing the Einstein equation in this way, we are essentially saying that the scale of a cosmological phenomenon - the curvature of spacetime - is determined by behaviour of matter at atomic scales. Not only is this counter-intuitive, but as we shall see, this assumption leads directly to the problems and paradoxes that currently beset cosmology and quantum gravity. Restating equation (12) in Planck units, by setting $G = c = 1$, does nothing to overcome this objection, assuming that we retain the direct relationship between the Planck scale and the atomic scale.

Returning to the standard derivation of the Friedman equations, since p is small in the present epoch, and ignoring for now the cosmological constant, (17) becomes

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = 0 \quad (18)$$

and (16) simplifies to

$$\frac{3\dot{a}^2}{a^2} + \frac{3k}{a^2} = 8\pi G\rho \quad (19)$$

One solution to equation (19) for a matter dominated universe is the Einstein-de Sitter model, with

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^{\frac{2}{3}} \quad (20)$$

It is conventional to define a critical energy density

$$\rho_{crit} = \frac{3H^2}{8\pi G} \quad (21)$$

and to express the energy content of the Universe as a fraction of this critical density

$$\Omega \equiv \rho/\rho_c$$

2.1.2. Age of the Universe From (16), and ignoring for now the cosmological constant, Λ , an expression can be derived for the age of the universe

$$t = \int_0^{a(t)} \frac{da}{\sqrt{8\pi G\rho_0 a^3(t_0)/3a(t) - kc^2}} \quad (22)$$

which, in the case of a flat universe with $k = 0$, evaluates to

$$t_0 = \frac{2}{3}H_0 \quad (23)$$

Current measurements of the Hubble constant from low-redshift Type Ia supernovae [10] give a value of $H_0 = 63.1 \pm 4.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Based on this same data, the standard Big Bang theory would (with $\Lambda = 0$) give a value for the age of the universe of $t_0 \approx 10 \times 10^9$ years, which is at odds with astrophysical and geological evidence.

2.1.3. Big Bang problems The standard cosmological problems are well documented in a number of sources, including [11]. We show here how they arise directly from the assumptions made in the preceding section, and also highlight an additional problem which has not previously been noted in the literature on this subject.

The horizon problem The standard theory is unable to explain how regions of the Universe that had not been in contact with each other since the Big Bang are observed to emit cosmic background radiation at almost precisely the same temperature as each other.

The flatness problem The fact that the observed matter density of the Universe is so close to the critical value necessary for the Universe to be closed implies that the ratio of these densities, Ω , must have been very close to one at the time of the Big Bang. This observation is the one of the main reasons for the widely held belief that there is probably additional hidden mass in the Universe such that Ω is in fact equal to one in the present epoch. The standard theory is unable to provide an explanation as to why the density of the Universe should be so close to the critical value.

The lambda problem Einstein originally included the cosmological constant Λ in his gravitational field equations in order to arrive at a solution that was consistent with the prevalent concept of a static Universe. The subsequent discovery that the Universe is in fact expanding has done nothing to diminish the enthusiasm on the part of many theorists for retaining this constant, in spite of Einstein's opinion that its inclusion was his 'biggest blunder'. Recent measurements of the expansion rate of the Universe [12] appear to suggest that a non-zero Λ may be causing the expansion to accelerate. However, the expansion of the Universe would have caused any initial cosmological constant to grow by a factor of 10^{128} since the Plank epoch. For Λ to be as small as it appears today presents yet another fine tuning problem.

The linearity problem For higher redshifts, the Hubble diagram can tell us whether the expansion of the Universe has undergone any periods of acceleration in its history. These results are derived using the expression for the expansion rate in terms of the redshift

$$H^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda \right] \quad (24)$$

where

$$\begin{aligned} \Omega_M &\equiv \left(\frac{8\pi G}{3H_0^2} \right) \rho_0 \\ \Omega_\Lambda &\equiv \frac{\Lambda}{3H_0^2} \\ \Omega_K &\equiv \frac{-k}{a_0^2 H_0^2} \end{aligned}$$

and

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

This leads to a formula for the lookback time in terms of the present value of the Hubble parameter, and the red-shift (see [9] for derivation)

$$t_0 - t_1 = H_0^{-1} \int_0^{z_1} (1+z)^{-1} \left[(1+z)^2 (1 + \Omega_M z) - z(2+z)\Omega_\Lambda \right]^{-\frac{1}{2}} dz \quad (25)$$

Analysis of the data obtained from high redshift supernovae [12] gives best fit values of $\Omega_M = 0.28$, $\Omega_\Lambda = 0.72$, and an age for the Universe of $H_0^{-1} \simeq 15 \times 10^9$ years. The values of the density parameters are remarkable in that they are very close to the values that would give a best fit curve corresponding to a linearly expanding universe of the same age, characterised by $t_0 = 1/H_0$. The fact that this is the case may merely be one

of the many coincidences that appear to exist in cosmology. Alternatively, it could be an indication that some underlying mechanism is in place that not only ensures that $\Omega_{Tot} = 1$, but also adjusts the cosmological constant over time such that $t_0 = 1/H_0$ will always apply.

2.2. Singularities

The standard formulation of the Einstein field equations leads inevitably to the concept of singularities: regions of space where the mutual gravitational energy of a body of matter becomes infinitely large, but is contained within an infinitesimally small volume. Standard Big Bang cosmology gives rise to singularities in at least two sets of circumstances: the initial Big Bang itself, and the interior of Black Holes. The origin of the Big Bang singularity follows directly from the dynamical equations described in Section 2.1. If the expansion of the Universe were to be reversed, then it is easy to see that there must have been a time when all the matter/energy in the Universe was contained in a vanishingly small volume of space. The proof that all Black Holes must contain a singularity is somewhat less intuitive, and was first achieved using topological methods in the Penrose-Hawking singularity theorem [13]. The concept of a singularity is abhorrent to most physicists, involving as it does the existence of infinities. Typically, the worst implications of singularities are avoided by invoking the Planck scale, with the assumption that the standard laws of physics somehow break down at energies greater than the Planck energy, or at distances less than the Planck length. However, this get-out is intellectually far from satisfactory. There is no obvious reason why a different set of physical laws should be applicable at the Planck scale. Nevertheless, the presumption that this is the case provides one of the main motivations for the study of Quantum Gravity.

2.3. Energy conservation

Mach's Principle, in its most basic form, asserts that the inertia experienced by a body results from the combined gravitational effects of all the matter in the Universe acting on it. Although a great admirer of Mach, Einstein was never entirely certain whether General Relativity incorporated Mach's Principle. Indeed, the issue is still the subject of continued debate even today (see [14] for example). A stronger version of Mach's Principle can be formulated, which states that the inertial mass energy of a matter particle is equal and opposite to the sum of the gravitational potential energy between the particle and all other matter in the Universe, such that:

$$mc^2 = - \sum_N \frac{Gm \cdot m}{r} \quad (26)$$

$$= - 4\pi\alpha Gm \int_0^R \rho(r)r^2 \cdot \frac{1}{r} dr \quad (27)$$

where $R \equiv c/H$ is the gravitational radius of the Universe, and α is a dimensionless constant. In the case of a homogeneous and isotropic matter distribution this becomes

$$mc^2 = -2\pi\alpha Gm\bar{\rho}R^2 \quad (28)$$

where $\bar{\rho}$ is the average matter density of the Universe.

Observational evidence suggests that the relationship $G\rho_0/H_0^2 \simeq 1$ is valid to a reasonable degree of precision in the present epoch. It would be particularly satisfying if this relationship were to be found to be true, as it would tie in with the concept that the Universe is ‘a free lunch’, i.e. all the matter in the Universe could be created out of nothing, with a zero net energy. However, it can be seen from (28) that rest mass energy appears to be proportional to $1/R$. Consequently, as the Universe continues to expand, the energy arising from mutual gravitational attraction will ultimately tend towards zero. Conversely, gravitational energy will become infinite at $t = 0$, the initial singularity. Since there is no suggestion that the rest mass energy associated with the matter in the Universe changes over time (unless one is considering the Steady State Theory), it would appear that this neat zero energy condition in the present era is just a coincidence.

2.4. The problem of time

The theories of Quantum Mechanics and classical General Relativity describe physics on vastly different energy and length scales. Nevertheless, they both have some underlying factors in common. For example, they are both based on the assumption that spacetime exists in the form of a four-dimensional differentiable manifold, with a Lorentzian metric structure. However, the role of time differs significantly between the two theories. This is not itself a problem when dealing with physics that lies solely within either of these domains. The problem of time really only manifests itself when attempts are made bring these two theories together into a theory of Quantum Gravity that can be applied to physics at the Planck scale. It is at this point that the difficulty reconciling the two different descriptions of time becomes apparent. The main arguments are described in some detail by Isham and Butterfield in [15], and an analysis of some aspects of the problem of time is given by Kuchař in [16]. In Quantum Mechanics, time plays an essential role as a background parameter in labelling the state of a quantum system; for example the time at which a measurement is made. Conversely, time in General Relativity is more closely integrated into the very structure of the theory itself. Essentially, time provides the fourth dimension of spacetime that allows a three-dimensional manifold to possess the property of curvature. Alternatively, time can be viewed as providing the offset between successive foliations of a three-dimensional manifold. Interestingly, neither the General Relativity nor the Quantum Mechanics description of time coincides closely with our own perception, which tends to view time as a linear flow of events with a clearly defined past, present and future. These concepts do not really exist at all in General Relativity time, and only do so to a limited extent in Quantum Mechanics.

3. Scale invariant gravity

The Einstein-Hilbert action of General Relativity contains an implicit assumption: that the gravitational force, which determines the behaviour of matter on a cosmological scale, has a fixed relationship with the fields and forces that govern the physics of matter on atomic scales. One can see how this assumption is compatible with a particle model of gravity, in which the gravitational force is mediated by the exchange of spin-2 gravitons. However, in any theory that treats gravity as a geometrical property of spacetime, this assumption appears to be seriously flawed. Indeed, it appears to be yet another example of anthropocentric thinking, as unreasonable in its own way as the pre-Copernican assertion that the Earth lies at the centre of the Universe. It may therefore be instructive to look at the consequences of breaking this linkage between the cosmological and quantum scales.

3.1. The Einstein equations revisited

If we are to do away with the Planck scale, then this immediately poses the problem of what to put in its place as the appropriate scale for dealing with gravitational phenomena. Arguably the simplest answer is to hypothesise that, since General Relativity describes the behaviour of the universe as a whole, the appropriate scale is that of the Universe itself. To make this work, it is helpful to define a universe as being a four dimensional manifold possessing the properties of compactness and closure. This in turn implies that the Universe is finite in extent. It is convenient, but not essential, to assume that the Universe has the topology of a 3-sphere. It is then possible to define a length scale in which the radius of the Universe R is taken to be unity, and the Gaussian curvature is given by $1/R^2 = 1$. Similarly, a universe is taken to have a unit mass, and to contain a finite number of matter quanta, N . For the sake of simplicity in the discussion that follows, it will be assumed that the matter content of the Universe is purely baryonic in nature. The mass of a single quantum of matter will therefore be given by $m = 1/N$ in these new units.

The basic principle underlying General Relativity is retained: that stress-energy induces curvature in the space-time manifold. However, we now rewrite the gravitational field equations in a form that follows directly from the definition of a universe given above, so that

$$G_{\mu\nu} = T_{\mu\nu} \tag{29}$$

where $T_{\mu\nu}$ is the stress-energy tensor in dimensionless normalised units, with the total stress-energy density of the universe as a whole being unity. If we wish to express the stress-energy tensor and curvature radius in more conventional units related to atomic measurement scales, then we need to apply appropriate re-normalisation factors. The curvature must be multiplied by $1/R^2$, where R is the radius of the Universe in our chosen length units, and the stress-energy tensor must be divided by the average

stress-energy density of the universe as a whole, $\bar{\rho}c^2$. The gravitational field equation now takes on the form

$$G_{\mu\nu} = \frac{3}{R^2 \bar{\rho} c^2} T_{\mu\nu} \quad (30)$$

It can be seen that by adopting this modified form for the gravitational field equations, that the space-time curvature arising from a given body of matter will depend on its scale relative to the Universe as a whole, and not solely on its mass. In the following section we shall see that this apparently minor change in definition has far reaching consequences.

It is worth noting that it is also possible to derive the modified gravitational field equation of (30) using Mach's Principle. This is achieved by using the energy conservation equation (28) to derive an expression for the gravitational constant G , which is then substituted in the standard Einstein equation (12). This approach is described in more detail in [17].

3.2. Revised cosmic dynamics

We look first at the implications of the modified gravitational field equations for the evolution of the Universe. The equations of motion are derived by combining (14,15) with the corresponding 00 and 11 components of the modified field equation (30) to give

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{c^2 \rho}{a^2 \bar{\rho}} \quad (31)$$

$$-\frac{2\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{kc^2}{a^2} = \frac{p}{\bar{\rho} a^2} \quad (32)$$

Since p is small in the present epoch, (32) becomes

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = 0 \quad (33)$$

and (31) simplifies to

$$\dot{a} = c \sqrt{\frac{\rho}{\bar{\rho}} - k} \quad (34)$$

From this equation it can be seen that when $\rho = \bar{\rho}$, and the curvature $k = 1$, the result is a quasi-static solution similar to the Einstein-DeSitter model, in that it has zero net energy and $\dot{a} = 0$. The solution is quasi-static in that the Universe is only flat and static in a cosmological reference frame. In regions of matter concentration where $\rho > \bar{\rho}$, k will effectively appear to be zero and the Universe will therefore appear to be expanding to an observer in this reference frame, with its horizon receding at the speed of light. In the remainder of this paper we shall refer to this as the Quasi-Static Universe (QSU) model.

This equation also embodies the negative feedback mechanism that ensures that ρ_{mat} will always be equal to ρ_{grav} . If at any point ρ_{mat} should exceed ρ_{grav} then this will

lead to a positive \dot{a} , which will tend to drive $\Omega \rightarrow 1$. The converse will apply if ρ_{mat} should fall below ρ_{grav} .

3.3. Newton's gravitational constant

In Section 3.1 a revised gravitational field equation (30) was constructed, which did not contain the Newtonian gravitational constant G that appears in the standard Einstein version of the equation. However, in the present epoch both these equations must reduce to the Newtonian case in the weak field limit, and must therefore be equivalent to each other:

$$\frac{3}{R^2 \bar{\rho} c^2} T_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (35)$$

giving

$$G = \frac{3c^2}{8\pi R^2 \bar{\rho}} \quad (36)$$

and since $t = 1/H$ in this model, we can express the gravitational 'constant' as

$$G = \frac{3H_0^2}{8\pi \bar{\rho}} \quad (37)$$

where $\bar{\rho}$ is the mean density of the Universe in standard atomic units. From this it is apparent that G is only a constant in the same sense that H_0 is a constant, i.e. for observers in the present epoch, as defined by atomic time. Since $\bar{\rho} \propto t^{-3}$ and $H_0^2 \propto t^{-2}$ we can see from (37) that $G \propto t$ in the atomic reference frame. The implications of a varying G will be discussed in Section 4.3. It is worth emphasising that the concept of a varying gravitational constant used here differs entirely from the phenomenological approach adopted in much of the existing literature on the topic of variable G (see [18] for example). The modified gravitational field equation described here is entirely free of any arbitrary constants when expressed in the appropriate natural cosmological units. The apparent gravitational constant that we observe is simply a consequence of our chosen length and mass scales relative to this cosmological scale.

3.4. Photon energy conservation

In Section 3.2 we saw that, in the reference frame appropriate to General Relativity, the Universe can be considered to be static. As such, the fourth dimension in General Relativity can best be viewed as being spacelike rather than timelike. If we now consider what happens to a photon that is emitted by some atomic process at a given point in time t_1 . At some later time t_2 an observer linked to the atomic scale (such as ourselves) would perceive that this photon had been redshifted due to the expansion of the universe in the time $t_2 - t_1$. Based on the assumption that the energy of the photon is directly proportional to its frequency, as given by the Planck formula $E = h\nu$, this observer would conclude that the photon had undergone an energy loss given by $E = h(\nu_1 - \nu_2)$. If, however, the photon is considered from the cosmological reference frame then it

does not undergo any change, since the universe is, by definition, static in this frame. The statement that the photon has lost energy is therefore seen to be invalid in this frame. In the absence of any relative motion between the cosmological frame and the atomic frame, the photon energy in the atomic frame must therefore also be unchanged. In other words, photons will retain the same energy that they originally possessed at the time of their emission, whether they are red-shifted (or blue-shifted) as a result of Doppler shift with respect to a given observer, climbing out of a gravitational potential, or being ‘stretched’ by the Hubble expansion of the Universe. The observable quantity that changes in the reference frame of the observer is the *power* of the photon.

Clearly, if this prediction is valid then it would have fundamental implications across many fields of physics. However the effects should be experimentally verifiable. In the field of cosmology, the most obvious place to look for evidence of photon energy conservation is in the cosmic microwave background (CMB). The standard theory is based on the assumption that as the Universe expands, the energy density due to matter decreases as $R^{-3}(t)$, whereas the energy density due to radiation decreases as $R^{-4}(t)$ because of the additional energy loss due to the red-shift. In the QSU model this remains true when looking at the spatial energy density. However, if one is carrying out measurements of photon energy by integrating power measurements over a period of time that is long in relation to the time span of the photon wavepacket (i.e. $t \gg \lambda/c$), then the energy will be found to decrease in proportion to $R^{-3}(t)$, as for the matter case. So, for example, a CMB photon emitted when the Universe had a temperature of $10^9 K$, with a present day temperature of $2.9 K$, would still retain its initial energy given by $h\nu = kT$, rather than $\sim 10^{-9}$ of this value. In [17] an experiment is proposed for determining photon energy from measurements of the CMB.

3.5. About time

One of the defining features of the Einstein gravitational field equation (12) is the fact that both sides of the equation are symmetric, divergence-free, second rank tensors. The stress-energy tensor $T_{\mu\nu}$ embodies the laws of energy and momentum conservation, such that $T^{\beta\alpha}_{;\alpha} = 0$. However, at first sight the RHS of the modified field equation in (37) would appear not to be divergence-free in that the expression that replaces the gravitational constant G seems to be time dependent, i.e.

$$\frac{\partial}{\partial x^0} \left(\frac{3c^2}{8\pi R^2 \bar{\rho}} \right) \neq 0 \quad (38)$$

It would appear that either we have to forego the divergence-free nature of the original Einstein equation, or we have to ‘adjust’ the new equation in some way in order to retain this desirable property. The latter approach was used in the Brans-Dicke scalar-tensor theory [1], and subsequently in the Canuto scale-covariant theory [7]. Although these fixes solved the immediate problem by restoring the zero-divergence property, this was done at the expense of the overall elegance of the solution, and ultimately its ability

to make any useful predictions. There is, however, another more radical solution to this problem.

In order to preserve the principle of general covariance, and still retain the features of the gravitational field equations, we must conclude that the period of an atomic clock is decreasing in proportion to the cosmological scale factor. Accordingly, we substitute for $R \equiv ct$ in (38) with $cn\tau$, where n is the time in atomic time units, and τ is the period of our atomic clock. Since $n \propto a = 1/\tau$, we see that the expression in (38) is invariant with respect to $x^0 \equiv \tau$, and hence possesses zero divergence.

This leads to a somewhat bizarre picture of time and space. From the perspective of an observer in the cosmological reference frame the Universe would appear to be static and closed. Any concentrations of matter in the Universe would appear to be shrinking in size. For an observer, such as ourselves, linked to the atomic reference frame, the Universe will appear to be flat and expanding, with the horizon receding at the speed of light. Our perception of time as a continuum is an illusion, and the time co-ordinate that we are used to is perhaps better described as subjective time. (This notion is not entirely novel; something similar was proposed by Jeans in 1931 [19]). The concept of two kinds of time - cosmological time and atomic time - is similar in some respects to the dual timescales postulated by Milne in his kinematic theory of gravity [21].

It is interesting to look at the conclusions that would be drawn by an experimenter who was somehow able to measure lengths using the cosmological scale appropriate to General Relativity, whilst still only having access to clocks based on atomic time. Being unaware that their clock was speeding up relative to cosmological time, such an observer attempting to measure the speed of light with sufficiently accurate apparatus would conclude that c was decreasing with time. This is analogous to the concept of varying speed of light cosmology (see [20] for example).

4. Consequences and Predictions

The definition of a Universe provided in Section 3, together with its associated gravitational field equations and resultant cosmic dynamics, describes what is essentially a new paradigm. Arguably, if this can be shown to be valid, then it would supplant the current standard model that includes the Big Bang theory and provides part of the rationale for pursuing a theory of Quantum Gravity. This section highlights some of the principal consequences that would follow from this new paradigm.

4.1. The Age of the Universe

The fact that this model describes a universe with a linearly increasing scale factor, such that $H = 1/t$, means that there is now no disparity between the measured value of the Hubble constant and the age of the universe as derived from astrophysical and geophysical data, i.e. about 15×10^9 years.

4.2. Solving the Big Bang problems

4.2.1. The flatness problem The dynamical equations in Section 3.2 clearly show that, for the Universe as a whole, the mean energy density is maintained at the critical value by a form of negative feedback mechanism. Under such circumstances, any small deviation of Ω from unity would result in an apparent acceleration or deceleration of the expansion rate so as to bring the system back to its equilibrium state.

4.2.2. The horizon problem The QSU model requires the Universe to be spatially closed, with the topology of a 3-sphere. Because the expansion rate is constant, the horizon distance will always be equal to the radius of the 3-sphere that defines the observable Universe. As a result of this coincidence of horizon distance and the radius of the observable Universe, all regions within the Universe will have been in causal contact with each other at time $t = 0$. This accounts for the observed homogeneity of the Universe, and provides an elegant solution to the horizon problem.

The smoothness problem, i.e. accounting for the perturbations in matter density that give rise to structure formation, is not explained directly by the QSU model. The variations in matter density that are required as a prerequisite for galaxy structure formation must therefore arise from some other mechanism.

4.2.3. The cosmological constant problem Since the QSU model accounts for the observed dynamics of the Universe without the requirement for a non-zero Λ , this constant can validly be omitted from the gravitational field equations. Hence the problem of how to explain a small, but non-zero, Λ disappears. Similarly, the linearity problem raised in Section 2.1.3 also ceases to be relevant as the QSU model results in a linearly expanding universe without the need to achieve a delicate balance between Ω_Λ and Ω_M . As was pointed out earlier, the scale factor of a universe with $\Omega_M \simeq 0.28$, $\Omega_\Lambda \simeq 0.72$ increases in a very similar way to the linear expansion predicted in Section 3.2. The main divergence between the two models occurs at redshifts in the range $0.8 < z < 1.8$. In [17] it was shown that results from the only supernova yet discovered in this redshift range tend to favour the QSU model over the standard model.

4.3. Variations in the gravitational constant

In Section 3.3 it was shown that Newton's gravitational constant, G , would be proportional to t in the reference frame of an observer on the atomic scale. The most accurate methods currently used to measure \dot{G}/G are generally based on the principle of measuring changes in planetary orbits within the solar system using radar ranging techniques. However, any calculation of \dot{G}/G based on distance measurements that are ultimately derived from atomic scale phenomena will generate a null result. This is due to the fact that time measured by any atomic or gravitational clock will be changing at the same rate as the distance to be measured. Suppose at time t_0 and

scale factor a_0 the measured distance is $2r_0 = cn\tau_0$, where τ_0 is the period of an atomic clock and n is the number of clock ticks between the emission of a radar signal and the reception of its reflection. At some future time when the scale factor has increased to a , the measurement is repeated. If $G(t) \propto a$ then the distance to the planet will have decreased so that $r = r_0 a_0/a$. However, the period of the atomic clock will also have decreased by the same proportion, with $\tau = \tau_0 a_0/a$. Consequently, the measured elapsed time for the radar signal round trip will still be n ticks, i.e. there will be no apparent change in distance and therefore no change in the calculated gravitational constant.

In order to verify that G does vary over cosmological timescales it is necessary either to measure its value directly using a Cavendish type experiment, or to turn to evidence from geophysical and astrophysical measurements. Since Cavendish experiments can currently only achieve accuracies of one part in 10^{-6} , these are not capable of detecting changes in G , which will be of the order of the Hubble factor, i.e. one part in 10^{-11} per year. We must therefore look to the other sources for indirect evidence of a time-varying G . Fortunately, a useful source of geophysical data does exist in the form of the Earth's fossil record, which can be used to track changes in the length of the Earth's day over geological timescales. Analysis of this data in [22] shows that the rotation of the Earth has slowed down since the planet was formed, due to tidal interaction with the Moon. Conservation of angular momentum of the Earth-Moon system dictates that there must be a corresponding increase in the orbital angular momentum of the Moon. Conventionally, this would be achieved by an increase in the radius of the Moon's orbit, and a corresponding decrease in the Moon's angular velocity, together with a lengthening of the sidereal month. However, observational data suggests that the Moon is in fact accelerating in its orbit, such that the change in the length of the lunar month is consistent with a time varying gravitational constant, with $G(t) \propto t$.

4.4. Primordial nucleosynthesis

Arguably, one of the few successes of the standard Hot Big Bang (HBB) model is its ability to predict the abundances of the light elements resulting from primordial nucleosynthesis. If the QSU model is to be of any use then it must also give predictions that are consistent with observational data. The QSU model implies that the observed power of the CMB is due to a relatively small number density of energetic photons rather than a very large number density of low energy photons. If we assume that the CMB photons originally had energies of $\sim 1\text{MeV}$, corresponding to a temperature of $\sim 10^{10}\text{K}$, and that these photons have been redshifted to the currently measured temperature of 2.7K , this implies an energy loss of the order of 10^{10} according to the standard theory. Since the currently observed CMB photon number density, calculated according to the standard theory, happens to correspond to $\eta_B \equiv n_B/n_\gamma \sim 10^{-10}$ baryons per photon, it follows that $\eta_B \simeq 1$ in the QSU model. This is an encouraging result in that it resolves the entropy problem: there is no a priori reason why the entropy of the universe should be as high as the value determined by the HBB model. However, it

presents a potential difficulty in reconciling observed light element abundances with an η_B of the order of unity.

According to the HBB model, the deuterium abundance in the Universe is particularly sensitive to the baryon-photon ratio, $\eta_B = 2.7 \times 10^{-8} \Omega_B h^2$. The HBB model requires a value of η_B corresponding to $\Omega_B h^2 \sim 0.02$ to give the observed Deuterium abundance of about 10^{-4} . However, it has been pointed out by Aguirre in [23] that the set of apparently arbitrary parameters that defines the standard HBB model, including η_B, Λ , etc, constitutes merely one point in parameter space that can lead to a universe compatible with human life. It is, in principle, possible that other combinations of the same parameters will also give rise to viable universes. As an example, the case of the classical Cold Big Bang (CBB) is examined. This model is characterised by a baryon-photon ratio of $\eta_B \simeq 1$. Detailed numerical simulations of primordial nucleosynthesis reactions [24] show that, with the appropriate values for the lepton-baryon ratio, η_λ , the CBB model is indeed able to create levels of metallicity that are compatible with those currently observed in the intergalactic medium.

The current CMB photon energy density is approximately 10^{-3} of the observed energy density due to baryonic matter. Taking these two observations together, this suggests that we are looking for a nucleosynthesis model that results in one photon per baryon, with an energy approximately 10^{-3} of the proton rest mass energy. The most obvious scenario is that of neutron decay, first proposed by Gamow [25]. Initial studies of the evolution of a cold neutron Universe have been carried out using numerical simulation models. These show that the initially cold, dense, neutron cloud heats up by means of β decay to form a hot proton-neutron-electron plasma at a temperature of $\sim 10^{11} K$. At $\sim 10^9 K$ a range of fusion reactions become energetically favorable, and lead to the formation of deuterium, tritium, helium and other light elements, as in the standard Big Bang nucleosynthesis models [26]. The main differences between the QSU model and the standard Big Bang is that in the former, photons play a negligible role in the exchange of energy between particles. The fact that the expansion rate is much slower also has a significant impact in that there is more time for fusion reactions to take place, and therefore an increased probability of synthesizing heavier elements than would be the case with the standard model. Nucleosynthesis in a linearly expanding Universe has also been studied by Lohiya in [27].

One of the most important consequences of nucleosynthesis in the QSU model is that the primordial baryonic matter is not able to cool down as the Universe expands, since there are insufficient photons to remove the entropy generated by the neutron decay and nuclear fusion processes. The primordial hydrogen and helium molecules will therefore remain in an ionized state indefinitely. This may explain why intergalactic gas clouds are currently observed to be ionized. Another feature of this model is that the β decay process that causes the primordial universe to heat up will give rise to scale-invariant differences in temperature, due to the statistical nature of the neutron decay reaction. Starting from a perfectly isotropic and homogeneous state, this mechanism is therefore able to account for the observed scale-invariant temperature fluctuations in

the CMB, which are explained by inflation in the standard Big Bang model.

4.5. The Large Number Hypothesis

A dimensionless quantity known as gravitational structure constant can be defined as the ratio of the electrostatic forces between two adjacent charged particles, e.g. protons, to the gravitational force between the particles.

$$\alpha_G = \frac{Gm_p^2}{\hbar c} \approx 5.9 \times 10^{-39} \quad (39)$$

Standard cosmological theories provide no obvious explanation for such a vast disparity between the forces of gravity and electromagnetism. In 1938 Dirac [28] noted that the dimensionless quantity $1/\alpha_G$ was approximately equal to the present age of the Universe measured in atomic time units (where 1 atomic time unit = $\hbar/m_p c^2 \approx 10^{-24}$ secs). If this relationship were to be valid for all epochs then this implies that $1/\alpha_G$ must be proportional to the age of the Universe, and therefore that $G(t) \propto 1/t$. This postulate formed the basis of Dirac's Large Number Hypothesis (LNH), which has subsequently provided the inspiration for a number of alternative cosmological theories. (It is worth noting that this formulation of the LNH is equivalent to the expression $G\rho_0/H_0^2 \simeq 1$ of Mach's principle).

In looking at some of the implications of this relationship for observers in the atomic reference frame it is helpful to express the mean gravitational energy density of the Universe in terms of the baryon number N , and the mean baryon mass, which we shall take to be the proton mass m_p . (Note that this implies, but does not require, that any missing mass in the Universe is baryonic in nature rather than in the form of other more exotic entities).

$$\bar{\rho} = \frac{3Nm_p}{4\pi R^3} \quad (40)$$

The expression for G in (36) can therefore be written as

$$G = \frac{R(t)c^2}{6Nm_p} \quad (41)$$

If we now substitute for G in equation (39) for the gravitational structure constant α_G , we find that

$$\alpha_G = \frac{R(t)cm_p}{6N\hbar} \quad (42)$$

where $R(t)$ is the apparent radius of the Universe in the atomic reference frame, at subjective time t . From this it can be seen that $\alpha_G \propto t$ in our reference frame, i.e. the strength of the gravitational interaction between particles will increase over time in relation to their mutual electromagnetic forces. Combining (42) with the expression for atomic time we find that

$$\alpha_G = \frac{n}{N} \quad (43)$$

where n is the time in atomic time units. Although this very simple result may at first seem somewhat surprising, it is perhaps to be expected, since the baryon number N is one of the few dimensionless quantities to occur naturally in cosmology. (The fact that $1/\alpha_G \approx n$ today is purely a coincidence).

4.6. Planck Units

We shall now examine the effects of recasting the expressions for the Planck units using the formula for G given in (41), and the de Broglie wavelength of a proton given by $R \approx \lambda_p \approx \hbar/m_p c$.

Planck Length Clearly with $G(t) \propto t$ the quantity known as the Planck Length in (2) will itself be a function of time such that $L_P(t) \propto \sqrt{t}$. Substituting for G using (41), and the expression for the atomic time unit, gives

$$L_P = \lambda_p \sqrt{\frac{n}{N}} \quad (44)$$

where n is the time expressed in atomic time units, and N is the baryon number of the Universe. It is interesting to note that at a time $n = N$ the Planck Length will have grown to a size such that $L_P = \lambda_p$. (Or conversely, in the cosmological frame, the proton wavelength will have shrunk below the Planck Length).

Planck Time A similar set of expressions can be derived for the quantity known as Planck Time in (3), to give

$$T_P = \frac{\lambda_p}{c} \sqrt{\frac{n}{N}} \quad (45)$$

Planck Mass From Equation (4) it is evident that the Planck Mass $M_P(t) \propto t^{-\frac{1}{2}}$. Again, substituting for G using (41), with $R = \lambda_p$ we find

$$M_P = m_p \sqrt{\frac{N}{n}} \quad (46)$$

And when $n = N$ we see that $M_P = m_p$.

Based on this analysis, the conclusion we must reach is that Planck Units do not represent a fundamental measurement scale that becomes relevant during the birth of the Universe and governs the realm of Quantum Gravity. Rather, they are scale factor dependent quantities which may shed some light on the behaviour of the Universe in its dying moments.

4.7. The ultimate fate of the Universe

In the preceding section we saw that at a time $n = N$ (where n is the time in atomic time units, and N is the baryon number of the Universe), the evolution of the scale invariant Universe reaches a state at which the Planck length is equal to the Compton

wavelength of the proton, and the Planck mass is equal to the proton mass. Recalling that the Schwartzchild radius of a black hole is given by

$$R_S = \frac{2GM}{c^2} \quad (47)$$

and substituting for G using (41), with $R(t) = N\lambda_p$ and $M = m_p$, we find that

$$R_S = \lambda_p \quad (48)$$

In other words, the scale factor of the Universe has evolved to the point where the radius of the proton exceeds the Schwartzchild radius corresponding to the proton mass. (The term proton here is used loosely to refer to whatever state baryonic matter may exist in the extreme gravitational conditions prevailing at this epoch. In practice it is more likely that protons and electrons will have recombined into atomic hydrogen by this stage, which in turn may have collapsed into neutrons in a reversal of the process described in 4.4 above). At this point the Universe effectively comes to an end as all protons simultaneously collapse into micro Black Holes - possibly to give birth to many more baby Universes according to Smolin in [29].

5. Conclusions and discussion

We have seen that, by challenging the assumption that spacetime curvature on a cosmological scale should be linked to the behaviour of fields on an atomic scale, it is possible to construct a gravitational field equation that is truly scale invariant. It has been shown that the cosmic dynamics resulting from this formulation of gravity describe a quasi-static universe, possessing a number of interesting properties. It is argued that the predicted behaviour of this QSU model provides a much more elegant explanation for a number of astronomical observations than does the standard HBB plus inflation model. Furthermore, the scale invariance inherent in the QSU model renders Newton's gravitational constant redundant, and with it the concept of a uniquely defined Planck scale. This feature in itself has a number of profound consequences across many areas of physics, some of which will be discussed here.

The elimination of the initial Big Bang singularity, taken together with the observation that the gravitational structure constant must be changing over time in the QSU model, leads to a very different picture of the early Universe. For example, at the epoch when matter fields initially come into existence, the electromagnetic self-energy of a particle will be exactly equal to the sum of the gravitational energy of the particle with respect to every other particle in the Universe. This should provide a useful clue about the nature of the mechanism that determines particle masses.

The concept of a uniquely defined Planck scale is one of the two principal motivations for the pursuit of a quantum theory of gravity, the other being the need for the curvature terms in the gravitational field equation to be quantized in order to be equivalent to the quantized matter fields. If the Planck factor is removed, we need to ask whether there is still a need for a theory of Quantum Gravity, at least in the form

currently being sought. There is no fundamental requirement that the gravitational field should have an inherent quantum structure, and it may well be more reasonable to think of the gravitational field as being quantised as a consequence of the matter fields with which it interacts.

The QSU model also has a number of implications for the role of time. The static nature of the Universe, when viewed from a cosmological reference frame, strongly suggests that at its most fundamental level the Universe does not possess any dynamical structure. If this were to be the case then we would need to reject the concept of time in canonical quantum gravity as being the separation between successive foliations of a three dimensional manifold. The fourth dimension in General Relativity would then take on a hyperspatial role, providing the additional dimension in Euclidean space in which the three dimensional manifold of the observable Universe can be curved. Time in Quantum Mechanics would continue to be a feature of the Lorentzian metric that overlays Euclidean hyperspace.

Finally, one can speculate that by more clearly defining the distinction between the realms of General Relativity and Quantum Mechanics, we can actually move closer towards constructing a paradigm that unifies the two theories.

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